First Estimations of Cosmological Parameters from Boomerang

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The anisotropy of the cosmic microwave background radiation contains information about the contents and history of the universe. We report new limits on cosmological parameters derived from the angular power spectrum measured in the Antarctic flight of the Boomerang experiment within the framework of inflation-motivated adiabatic cold dark matter models. Using a weak prior probability on the Hubble expansion parameter h we find the curvature is close to flat and the primordial fluctuation spectrum is nearly scale invariant, in agreement with the basic inflation paradigm. We find that the data prefer a baryon density $\Omega_b h^2$ above, though similar to, the estimates from light element abundances and big bang nucleosynthesis. When combined with large scale structure observations, the Boomerang data provide evidence for both dark matter and dark energy contributions to the total energy density $\Omega_{tot}$.

The angular power spectrum $C_\ell$ of temperature anisotropy in the cosmic microwave background (CMB) is a powerful probe of the content and nature of the universe. The DMR instrument on the COBE satellite measured $C_\ell$ for multipoles $\ell \lesssim 20$, corresponding to angular scales larger than $\sim 5$º[1]. Significant experimental effort by many groups focusing on smaller angular scales, when combined [2–4], led to the $C_\ell$ estimates in the $\ell$ bands marked with closed circles in Figure 1, which indicate a peak at $\ell \sim 200$. It has long been recognized that if $C_\ell$ can be determined with high precision over these angular scales, parameters such as the total energy density and baryon content of the universe, and the shape of the primordial power spectrum of density fluctuations, could be accurately measured [5]. The most recently published Boomerang angular power spectrum shown in Figure 1 represents a qualitative step towards such high precision [7] (hereafter, B98).

The data define a strong peak at $\ell \sim 200$. The steep drop in power from $\ell \sim 200$ to $\ell \sim 400$ is consistent with the structure expected from acoustic oscillations in adiabatic cold dark matter (CDM) models of the universe, but is not consistent with locations and widths of peaks expected in the simplest cosmic string, global topological defect, and isocurvature perturbation models [8]. The data at higher $\ell$ also show strong detections which limit the height of a second peak, but are consistent with the height expected in many CDM models.

In this paper, we concentrate on determining a set of 7 cosmological parameters that characterize a very broad class of CDM models by statistically confronting the theoretical $C_\ell$'s with the B98 and DMR bandpower data. Sample CDM models that fit the data are shown in Figure 1. These are best-fit theoretical models using successively more restrictive “prior probabilities” on the parameters, a theme of this paper. Some of these priors are
quite weak and are generally agreed upon by all cosmologists, for example that the Universe is older than 10 Gyr and that the Hubble constant $H_0 = 100\ h\ km\ s^{-1}\ Mpc^{-1}$ lies between 45 and 90. Stronger priors rely on specific measurements, e.g., the HST key project determination of $H_0$ to 10% accuracy [9] and the determination of the cosmological baryon density, $\omega_b \equiv \Omega_b h^2$, to 10% [10]. In [7], we applied a medium set of priors to the B98 power spectrum to constrain a 6 cosmological parameter model and found a 95% confidence limit for $\Omega_m$ of $0.88 < \Omega_m < 1.12$. Row P0 of Table I shows the result for our full 7 parameter set with a similar medium prior (here taken to be $h = 0.65 \pm 0.1, \omega_b = 0.019 \pm 0.006$, with Gaussian errors for both). As we progress through the Table, we show the effect of either weakening or strengthening the prior from this starting point.

Two of our parameters are fundamental for describing the physics of the radiative transport of the CMB through the epoch at $z \sim 1100$, when the photons decoupled from the baryons. These are $\omega_b$ and the CDM density $\Omega_c$. The acoustic patterns at decoupling are related to the sound-crossing distance at that time, $r_s$, which is sensitive to these parameters. We fix the density of photons and neutrinos [11], which are other important constituents at this epoch. The observed B98 patterns are also sensitive to the “angular diameter distance” to photon decoupling, mapping the $z \sim 1100$ spatial structure to the angular structure, and, through its dependence on geometry, to $\Omega_m$, the total energy in units of the critical density. When $\Omega_m < 1$ (open models), $r_s$ is mapped to a small angular scale; when $\Omega_m > 1$ (closed models), $r_s$ is mapped to a large angular scale.

This mapping also depends upon the density associated with a cosmological constant, $\Omega_\Lambda$, and $\Omega_m \equiv (\omega_c + \omega_b)/h^2$. Combinations of $\Omega_m$ and $\Omega_\Lambda$ which give the same angular diameter distance will give nearly identical CMB patterns, resulting in a near degeneracy that is broken only at large angular scales where photon transport through time-varying gravitational potentials plays a role. One implication of this is that $\Omega_\Lambda$ cannot be well determined by our data alone, in spite of the high precision of B98. We have paid special attention to such near-degeneracies [12] throughout our analysis.

The universe reionized sometime between photon decoupling and $z \sim 5$. This suppresses $C_l$ at small scales by a factor $e^{-2\tau_C}$, where $\tau_C$, our fifth parameter, is the optical depth to Thompson scattering from the epoch at which the universe reionized to the present.

Our last two parameters characterize the nature of the fluctuations arising in the early universe, through a power law “tilt” $n_s$ and an overall amplitude factor for the primordial perturbations. The simplest inflation models have a nearly scale invariant spectrum characterized by $n_s \approx 1$. Of course, many more variables, and even functions, may be needed to specify the primordial fluctuations, in particular those describing the possible contribution of gravity waves, whose role we have also tested [13]. For our overall amplitude parameter, we use $\ln C_{10}$ where $C_{10}$ is the CMB power in the theoretical spectrum at $\ell = 10$. If we wish to relate the CMB data to large scale structure observations of the Universe (LSS), we use $\ln \sigma_8^2$ as the amplitude parameter, where $\sigma_8^2$ is the model power in the density fluctuations on the scale of clusters of galaxies ($8 h^{-1} \text{Mpc}$).

Our adopted parameter space is therefore $\{\omega_b, \omega_c, \Omega_m, \Omega_\Lambda, n_s, \tau_C, \ln C_{10}\}$. The amplitude $C_{10}$ is a continuous variable, and the rest are discretized for the purpose of constructing the model database we use to compare data and theory. The number of values and coverage are: 15, over 0.1 $\leq \Omega_m \leq 1.5$; 14, over 0.0031 $\leq \omega_b \leq 0.2$; 10, over 0.03 $\leq \omega_c \leq 0.8$; 11, over 0 $\leq \Omega_\Lambda \leq 1.1$; 9, over 0 $\leq \tau_C \leq 0.5$; 31, over 0.5 $\leq n_s \leq 1.5$. The spacings in each dimension are uneven, designed to concentrate coverage in the regions preferred by the data and yet still map the outlying regions. Fast CMB transport programs [14] were used to construct our $C_l$ databases. Use was made of the angular-diameter distance degeneracy and $\ell$-space compression to reduce the size and computational requirements needed to construct such a database. In the course of this work, we have also explored a number of other parameter space choices; for example a finely gridded one using $\{\Omega_c, \Omega_b, h\}$ in place of $\{\Omega_m, \omega_b, \omega_c\}$. This allowed us to test the influence of discretization and different variable combinations on our results. We find excellent agreement between the results of the different parameterizations.

Parameter estimation is an integral part of the B98 analysis pipeline, which makes statistically well-defined maps and corresponding noise matrices from the time-ordered data, from which we compute a set of maximum likelihood bandpowers, $C_B$. The likelihood curvature matrix $\mathcal{F}_{BB'}$ is calculated to provide error estimates including correlations between bandpowers. The curvature matrix $\mathcal{F}_{BB'}$ and the curvature matrix evaluated at zero signal, $\mathcal{F}^0_{BB'}$, are used in the offset-lognormal approximation [2] to compute likelihood functions $L(x, \tilde{y}) = P(\tilde{C}_B | x, \tilde{y})$ for each combination of parameters $x$ and $\tilde{y}$ in our database. Here $x$ is the value of the parameter we are limiting, $\tilde{y}$ specifies the values of the other parameters.

We multiply the likelihood by our chosen priors, and marginalize over the values of the other parameters $\tilde{y}$, including the systematic uncertainties in the beamwidth and calibration of the measurement [15]. This yields the marginalized likelihood distribution

$$L(x) \equiv \int P(x, \tilde{y})L(x, \tilde{y})d\tilde{y}. \quad (1)$$

For clear detections, central values and $1\sigma$ limits for the explicit database parameters mentioned above are found from the $16\%$, $50\%$ and $84\%$ integrals of $L(x)$. When no clear detection exists, these errors can be misleading, so for these cases we shift to likelihood falloffs by $e^{-1/2}$ from the maximum, or variances about the mean of the distribution $L(x)$. The mean and variance are used to
set the limits on other “auxiliary” parameters such as $h$ and $\Omega_m$, which may be nonlinear combinations of the database variables. For good detections the three estimation methods give very good agreement, and yield 2σ errors that are roughly twice the 1σ ones generally reported in this paper.

We have used this method to estimate parameters, using the B98 power spectrum of Figure 1 with the COBE bandpowers determined by [2] and a variety of priors. The results are summarized in Table I; likelihood functions for selected parameters and priors are shown in Figure 2.

In the presence of degenerate and ill-constrained combinations of parameters, as with CMB data, the edges of the data-base form implicit priors. We have constructed our database such that these effective priors are extremely broad. This allows us to probe the dependence of our results on individually imposed priors. The choice of measure on the space is itself a prior; we have used a linear measure in each of our variables [16]. Priors can break parameter degeneracies and result in more stringent limits on the cosmological parameters. Artificially restrictive databases or priors can lead to misleading results; thus, priors should be both well motivated and tested for stability. We therefore regard it as essential that the role of “hidden priors” in any choice for $C_l$ database construction be clearly articulated.

We now discuss the results of this exercise, in the general order of weakest to strongest applied priors.

Our “entire database” analysis prefers closed models with very high $\omega_b$, as shown in line P1 of Table I and in Figure 2. The low sound speed of these models couples with the closed geometry to fit the peak near $\ell \sim 200$. These models require very high values of $h$ and $\omega_b$, and have extremely low ages, so we have mapped out this region using a coarse grid. Applying weak priors (lines P2-P4 in Table I) moves the result away from those models, to a regime which is stable upon application of further priors and analysis with smaller databases, as shown in panels 1 (top left) and 4 (bottom left) of Figure 2. Given their gross conflict with multiple other cosmological tests we do not advocate the “entire database” models as representative of the actual universe, and we proceed with prior-limited analyses below.

The analysis with weak priors (P2-P4) finds that the curvature is nearly flat, while favoring slightly closed models. The migration toward $\Omega_m = 1$ as additional priors are applied, as shown in Table I and panel 1 of Figure 2, suggests caution in the interpretation of 1σ effects. The baryon density $\omega_b$ is also well constrained. While our results are in formal statistical disagreement with the $\omega_b$ estimates from light element abundances [10], it is none the less remarkable that our entirely independent method yields a result that is so close. The scalar spectral index $n_s$ is very stable once weak priors are applied, and is near the value expected from inflation. This weak prior analysis does not yield a significant detection of $\Omega_L$; the $\Omega_b h^2$ results in Table I are suggestive of a detection, but are at least in part driven by the weak priors acting on limits of the database [17, 18]. The values of $\tau_C$ are in the range of expectation of the models, but there is no clear detection.

In row P4a, we add a “CMB prior”, which is actually a full likelihood analysis of all prior CMB experiments combined with B98 and DMR, including appropriate filter functions, calibration uncertainties, correlations, and noise estimates for use in the offset-lognormal approximation [2]. As would be expected given the errors we compute on the compressed bandpowers of these experiments in Figure 1 cf. those for B98, this CMB prior only slightly modifies the B98-derived parameters, with $n_s$ the most notable migration. None the less, as much previous analysis of the prior heterogeneous CMB datasets has shown [19], reasonably strong cosmological conclusions could already be drawn on $n_s$ and $\Omega_m$. Row P4b shows results for the weak prior...
case of excluding B98, through our machinery. Though
$n_s$ and $\Omega_k$ have detections consistent with the B98 re-
sults, no conclusions can be drawn on $\omega_b$ (though the
whole database does pick up the high $\omega_b$, $\Omega_{tot}$ region.)
We note that if $\tau_C \approx 0$ is enforced, most variables re-
main unmoved, but $n_s$, which is well-correlated with $\tau_C$,
moves closer to unity: for P4,P4a,P4b, we would have
$n_s = 0.97, 0.99, 1.04$, respectively, and for P5, P5a,P5b,
we would have $n_s = 0.94, 0.96, 0.99$. A prior probability
of $\tau_C$ based on ideas of early star formation would help
to decrease the $n_s$ degree of freedom.

The $\Omega_{tot}$, $\omega_b$, and $n_s$ results are stable to the addition
of a prior which imposes two constraints derived from
large scale structure (LSS) observations [6]. The first is
an estimate of $\sigma^2_\delta$ that requires the theory in question
to reproduce the local abundance of clusters of galax-
ies. The second is an estimate of a shape parameter for
the density power spectrum derived from observations of
galaxy clustering [20]. Adding LSS to the weak $h$ and
BBN priors (P5, and panels 2 (top center) and 3 (top
right) of Figure 2) breaks a degeneracy, yielding a weak
determination of $\Omega_\Lambda$ that is consistent with “cosmic con-
cordance” models. This also occurs when LSS is added
to only the prior CMB data (P5b and [6]). The LSS prior also
strengthens the statistical significance of the deter-
mination of $\Omega_k h^2$. Panel 3 of Figure 2 shows likelihood
contours in the $\Omega_k = 1 - \Omega_{tot}$ vs. $\Omega_\Lambda$ plane. Here we
have plotted the LSS prior (P5), which begins to localize
the contours [21] away from the $\Omega_\Lambda = 0$ axis, toward a
region that is consistent with the SNIa results [22].

The use of a strong $h$ prior alone yields results very
similar to those for the weak $h$ case. The strong BBN
prior, however, shifts many of the results from the weak
BBN case. Our data indicate a higher $\Omega_k h^2$ than BBN,
and constraining it with the BBN prior shifts the values
of several parameters, including $\Omega, h^2$, $\Omega_\Lambda$, $n_s$, and $\Omega_m$.
Additional “strong prior” results (P8-P11) are shown in
Table I, as an exercise in the power of combining other
constraints with CMB data of this quality.

A number of the cosmological parameters are highly
correlated, reflecting weak degeneracies in the broad but
restricted $\ell$-space range that the B98+DMR data cov-
erS [12]. Some of these degeneracies can be broken with
data at higher $\ell$, as is visually evident in the radically dif-
ferent behavior of the models of Figure 1 beyond $l \sim 600$.
To understand the degeneracies within the context of this
data, we have explored the structure of the parameter co-
variance matrix $\langle \Delta y_i \Delta y_j \rangle$, both for the database param-
eters and the ones derived from them. They add motiva-
tion for the specific parameter choices we have made [23].

Parameter eigenmodes [5, 12] of the covariance matrix,
found by rotating into principal components, explicitly
show the combinations of physical database variables
that give orthogonal error bars. A by-product is a rank
ordered set of eigenvalues, which show that for the cur-
rent B98 data, 3 combinations of the 7 parameters are
determined to better than 10% [24].

The B98 power spectrum reported in Figure 1 was cal-
culated from a coarsely pixelized map made by a single
channel over a limited region of the B98 sky coverage. By
using more of the B98 data and making maps with finer
pixelization, we hope to reduce the errors and extend the
bandpower estimates to higher $\ell$. Such work, and the
estimation of the cosmological parameters it implies, is a
primary goal of future analyses of BOOMERANG data.
TABLE I: Results of parameter extraction using successively more restrictive priors. The confidence intervals are 1σ, evaluated using methods described in the text. The 2σ errors are approximately double the 1σ values quoted in most cases. The quoted values are reported after marginalizing over all other parameters. Note that these combinations are not, and should not be, the parameters of the “maximum likelihood” best-fit models of Figure 1. The weak h and BBN (Ω₉h²) priors are tophat functions (uniform priors) and both include an additional age > 10 Gyr prior. The strong priors are Gaussians with the stated 1σ error. P0 is the medium h + BBN prior used in [7] and described in the text. The LSS priors are combinations of Gaussians and tophats [20]. Rows P4a and P5a show the small effect of including prior CMB data in our B98+DMR analysis; these should be contrasted with P4b and P5b, the case of prior CMB data alone. Columns 1-5 (Ω₀₉, Ω₀h, ν₀, Ω₃, τ) are predominantly driven by the CMB data, except for Ω₉h² and Ω₀h when the strong prior prior (P7-P9) is applied. Most of the values in columns 6-10 (τc to Age) are influenced by the structure of the parameter space and should not be interpreted as CMB-driven constraints; exceptions are the Ω₀h² results when the LSS prior is applied. An equivalent table that includes an inflation-inspired gravity wave induced contribution to the anisotropy [13] yields remarkably similar parameters and errors.

<table>
<thead>
<tr>
<th>Priors</th>
<th>Ω₀₉</th>
<th>Ω₀h</th>
<th>ν₀</th>
<th>Ω₃</th>
<th>τ</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0: Whole Database</td>
<td>1.30</td>
<td>0.92</td>
<td>0.49</td>
<td>0.05</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>P1: Medium h + BBN</td>
<td>1.60</td>
<td>0.55</td>
<td>0.05</td>
<td>0.05</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>P2: Weak h (0.45 &lt; h &lt; 0.90)</td>
<td>1.51</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P3: Weak BBN (Ω₀h² &lt; 0.038)</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P4: Weak h+BBN</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P4a: Weak &amp; prior CMB</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P4b NO B98: Weak &amp; prior CMB</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P5: LSS &amp; Weak h+BBN</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P5a: LSS &amp; Weak &amp; prior CMB</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P5b NO B98: LSS &amp; Weak &amp; CMB</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P6: Strong h (h = 0.71 ± 0.08)</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P7: Strong BBN (Ω₀h² = 0.019 ± 0.002)</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P8: Strong h+BBN</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P9: LSS &amp; Strong h+BBN</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P10: Ω₀₉ = 1 &amp; Weak h+BBN</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>P11: Ω₀₉ = 1 &amp; LSS &amp; Weak h+BBN</td>
<td>1.30</td>
<td>0.10</td>
<td>0.08</td>
<td>0.05</td>
<td>0.10</td>
<td>0.14</td>
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[11] The C_l spectra with massive neutrinos are quite similar to those without, and current data, including B98, will not be able to strongly constrain the value. When the LSS prior is added to the CMB data, however, the combination is quite powerful, e.g., [6].
[13] Gravity waves (GW) can induce CMB anisotropy, and could have a separate tilt, n_l, and an overall amplitude. They have little effect over the range of l's that B98 is most sensitive to, but could have an important impact.

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[13] Gravity waves (GW) can induce CMB anisotropy, and could have a separate tilt, n_l, and an overall amplitude. They have little effect over the range of l’s that B98 is most sensitive to, but could have an important impact.
on the amplitude relative to COBE. To test the role that
GW induced anisotropies would play, we have adopted
the model used by [6]: for \( n_s < 1 \), we set \( n_s = n_s - 1 \) and
for \( n_s > 1 \), we allow no GW contribution. This presents
a fixed alternative, reasonably motivated by inflation,
without introducing new parameters. We have found that
there is a negligible effect on the parameter determina-
tions in Table I; there is only a very slight migration
upward in \( n_s \).


[15] Apart from the 7 stated database parameters, we have
allowed for an estimated 10% uncertainty in the calibra-
tion and the beam, which we determine simultaneously
with the overall amplitude \( C_{10} \), by relaxing to the maxi-

mum likelihood value in these variables. We then deter-
mine the Fisher error matrix, assume that the variables
are log-normally distributed, and evaluate a correction to
the likelihoods appropriate for marginalization over these
“intrinsic” variables. Including the marginalization cor-
rection makes little difference. We have also marginalized
over bins that were used in creating the power spectrum
but not in the analysis, since they are correlated.

[16] The choice of measure is not important for strong loc-
alyzed peaks, but can potentially affect limits on poorly
constrained variables and on those with complex likeli-
hood functions. One can argue for logarithmic measures
in \( C_{10} \) (as we have used) and in \( \omega_h \) and \( \omega_s \) (which we
have not used), and there are certainly philosophical al-
ter natives to linear measures in \( \Omega_{b0} \) and \( \Omega_A \). Consider
what happens when we turn the “whole database” \( \omega_b \)
likelihood curve of Figure 2 into a probability curve if we
adopt a logarithmic rather than linear measure: the
anomalous peak at 0.1 drops below the “cosmological peak”
at 0.03, once weak priors are adopted, the 0.03 peak is all that is left and it is very stable to changing
the measure. Changing measures usually moves peaks a
small fraction of a \( \sigma \), although the amount does depend
upon relationships to correlated variables with large
errors. The discreteness of our database is also a restric-
tion on how accurately we can localize peaks. For
example, our finest gridding in \( \Omega_{b0} \) is 0.05 from 0.8 to 1.2,
hence accurate localization better than half this spacing
should not be expected. When projections are made, the
available volume of models leads to effective priors as
well [17, 18].

[17] The weak prior by itself actually focuses \( \omega_c \) about 0.22,
dropping to either side because of \( h \) and age restrictions.
Our data do constrain \( \omega_c \) further, but not enough to
claim a CMB determination beyond the prior until the
LSS prior is included.

[18] \( \Omega_A \) and \( \Omega_{b0} \) have a prior probability dropping as \( \Omega_{b0} \)
drops and \( \Omega_A \) rises just because \( \Omega_{b0} \) is positive. There is
a physical effect that also favors the closed models when
CMB is added. As \( \omega_b \) varies, the sound speed lowers, the
peak moves to higher \( \ell \), but can be mapped back to our
observed \( \ell_{pk} \) by judiciously choosing an \( \Omega_{b0} > 1 \). \( \Omega_A > 0 \)
moves the peak to lower \( \ell \) which \( \Omega_{b0} < 1 \) can also move
back to the observed \( \ell_{pk} \), but it is a smaller effect. If we
had all low \( \Omega_A \) < 0, closed models could have done the
same, further favoring \( \Omega_{b0} > 1 \) because of the volume of
models available.

[19] e.g., M. White et al., Mon. Not. R. Astron. Soc. 283,
pp107, (1996); K. Gang et al., Astrophys. J. 484, 7
[6]; G. Efstathiou et al., Mon. Not. R. Astron. Soc. 303,
84, 3523 (2000); A. Melchiori et al., submitted to
(2000); M. Le Dour et al., submitted to Astron. and As-

[20] The LSS prior is a slight modification of the one
used in [6]. \( \sigma_\Omega^2_{m,50} = 0.55^{+0.02}_{-0.08} \) is assumed to be
distributed as a Gaussian smeared by a tophat distribu-
tion, with the first error indicating the 1-\( \sigma \) error on
the Gaussian, and the second indicating the extent of
the tophat about the mean. The constraint from power
spectrum shapes involves a combination of spectrum
tilt, \( n_s - 1 \), and a “scaling shape parameter” \( \Gamma \approx \Omega_m h e^{-(\Omega_5e_{-5} + \Omega_5^{-5/2})^{-5/6}} \) which is related to the
horizon scale when the Universe passed from relativistic
to matter dominance: \( \Gamma + (n_s - 1)/2 = 0.22^{+0.07}_{-0.08} \).
Both priors were designed to generously encompass the ob-
servations, and so are “weak” to “medium” rather than
“strong”, in the sense of Table I.

[21] The contours plotted at \( L / L_{max} = \exp[-(1, 4, 9)/2] \)
provide only rough indicators of 1, 2, and 3\( \sigma \); less restric-
tive estimates appropriate to symmetric errors in the two
variables are also often plotted [12].

[22] Riess et al., Astron. J. 116, 1099 (1998); S. Perlmutter

[23] Here are some sample correlation coefficients for the weak
+BBN case of Table I: it is relatively small between \( \omega_b \)
and other database variables but between \( \Omega_b \) and \( h \) it
is 90%. Similarly, as is evident from the contour map
in Figure 2, \( \Omega_b \) and \( \Omega_A \) are correlated only at the 35% level,
whereas \( \Omega_m \) and \( \Omega_A \) are correlated at the 97% level.
Thus, for CMB work it is advantageous to use \( \Omega_b \) as a
variable rather than \( \Omega_m \), and hence this is what we plot
in Figure 2 rather than the more recognizable \( \Omega_m \)-\( \Omega_A \)
plot. \( \omega_c \) and \( \Omega_A \) have a 76% correlation, not surprising
in view of that for \( \Omega_m \).

[24] For the P4 case, the best determined eigenmode (to
\( \pm 0.03 \)) is a combination of slope, amplitude and Thomp-
son depth; next (to \( \pm 0.04 \)) is predominantly \( \Omega_b \), with a
judicious negative contribution from \( \Omega_A \), a combination
orthogonal to the angular diameter distance degeneracy;
the third eigenmode (to \( \pm 0.08 \)) is mostly \( \omega_b \), with a lit-
tle contribution from all other variables. The next three
combinations are determined to \( \pm 0.15 \); the worst (\( \pm 0.4 \))
combination is one of \( \omega_c \) and \( \Omega_A \). Similar coefficients
and accuracies hold for other priors, except for distortions
in the strong BBN prior case.
FIG. 2: Likelihood functions for a subset of the priors used in Table I. Panel 1 (top left) shows the likelihood for \( \Omega_k \equiv 1 - \Omega_{\text{tot}} \); the full-database (P1, dotted line) prefers closed models, but reasonable priors (P2, dashed line; P4, solid line) progressively move toward \( \Omega_k = 0 \). The medium prior P0 of [7] is dot-dashed. We caution the reader against aggressively interpreting any 2\( \sigma \) effects. Likelihood curves for \( \Omega_\Lambda \) are shown in panel 2 (top center). In panels 2 and 4-6, the cases and line types are as in panel 1, except that dot-dashed now denotes the weak+LSS prior, P5. With weak priors applied, no significant limit on \( \Omega_\Lambda \) is set (P2, dashed line; P4, solid line in all remaining \( \mathcal{L}(x) \) panels). Only by adding the LSS prior is \( \Omega_\Lambda \) localized away from zero (P5, dot-dash in all remaining \( \mathcal{L}(x) \) panels). Panel 3 (top right) shows the contour plot of \( \Omega_h \) and \( \Omega_\Lambda \), for which the first two panels are projections to one axis. The bold diagonal lines mark \( \Omega_m=1 \) and \( \Omega_m=0 \). The bold (blue) contours are those found with the LSS prior (P5), plotted at 1, 2, and 3\( \sigma \). SN1a constraints are similarly plotted as the lighter (black) smooth contours, and are consistent with the CMB contours at the 1\( \sigma \) level. When the SN1a prior is applied as well, the result is the light (red) contours, localized near \( \Omega_\Lambda \sim 0.7 \). Panel 4 (bottom left) shows the contours for \( \omega_\Lambda \); the full database analysis results in a bimodal distribution with the higher peak concentrated at very high values. These high \( \omega_\Lambda \) models are eliminated by the application of a weak \( h \) prior (P2, dashed). Even the weak BBN prior (P4, solid) clearly overconstrains the data. Panel 5 (bottom center) shows a localization of \( \omega_\Lambda \) for the weak \( h \) and BBN prior cases, but this is partially due to the effect of the database structure coupling to the \( h \) and age priors. Only the LSS prior (P5, dot-dash) allows the CMB to significantly constrain \( \omega_\Lambda \). Panel 6 (bottom right) shows good localization and consistency in the \( n_s \) determination once any priors are applied. The inflation-motivated \( \Omega_{\text{tot}}=1 \) priors (P10, P11) give very similar curves localized around unity.