

Space carriers

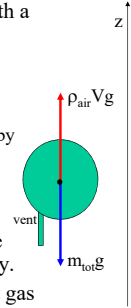
- Reaching space altitudes is not easy.
 - Stratospheric balloons can carry a payload at an altitude of about 40 km for 1-30 days.
 - Sounding rockets can carry a payload at an altitude of 400 km (for a few minutes)
 - Satellites are put in orbit by larger rockets, at altitudes > 400 km
 - The Space station is orbiting at an altitude of 400 km
 - Payloads can be put (using very large rockets) in non-Earth orbits (L2, Moon) or sent into interplanetary missions.
- In the following we give a short summary of the physical principles of stratospheric balloons and rocket propulsion.

Stratospheric Balloons

- A balloon is a light bag, volume V , filled with a gas lighter than the surrounding air.
- The forces acting on the balloon are
 - Gravity, due to the mass of balloon, gas, and payload; directed downwards
 - Bouyancy, equal to the weight of the air moved by the balloon volume V ; directed upwards.

$$F_z = -m_{\text{tot}}g + V\rho_{\text{air}}g$$

- The balloon will lift until $F_z=0$, i.e. until the Archimedes lift will fully compensate gravity.
- If the balloon is open at the bottom, the light gas can vent and the pressure inside the balloon is always equal to the surrounding air pressure.



Stratospheric Balloons

- The pressure where equilibrium is reached (at float altitude) can be found as follows:

$$0 = -m_{\text{tot}}g + V\rho_{\text{air}}g =$$

$$= -m_{\text{payl}}g - m_{\text{ball}}g - V\rho_{\text{gas}}g + V\rho_{\text{air}}g$$

- Using perfect gases law: $\rho_x = \frac{M_x p}{RT}$

(where M_x is the molar weight of gas x , and $R=0.0821$ (lit atm / K mol)) we get

$$p_{\text{float}} = \frac{RT}{V} \frac{m_{\text{payl}} + m_{\text{ball}}}{M_{\text{air}} - M_{\text{gas}}}$$

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- For a given payload mass, the float pressure will be lower (and the float altitude higher) for high values of the ratio

$$\frac{V}{m_{\text{ball}}} = \frac{\frac{4\pi}{3} R^3}{\rho_{\text{ball}} 4\pi R^2 t} = \frac{R}{3\rho_{\text{ball}} t}$$

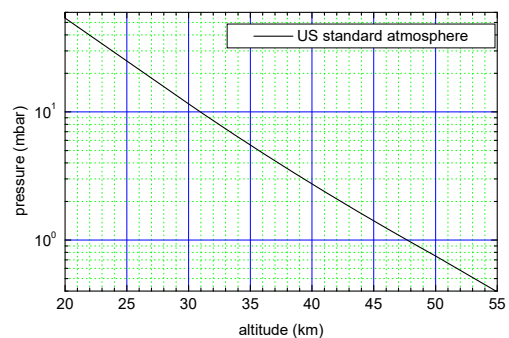
- It is evident that we need a large balloon, made of a thin, low density material. For stratospheric balloons, polyethylene (density around 1; basically the same as trash bags) with fiberglass reinforcements is used. The polyethylene layer is very thin ($t=15$ to $25 \mu\text{m}$); the volumes range from 5000 to $2 \times 10^6 \text{ m}^3$; consequently m_{ball} ranges from 30 Kg to 1500 Kg .

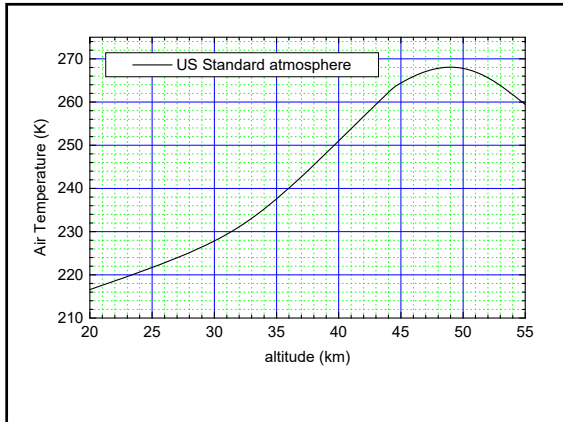
Example 1

- Assume a 10^6 m^3 balloon (10^3 Kg mass) lifting a 10^3 Kg payload. The balloon is inflated with He ($M_{\text{He}}=4 \text{ g/mol}$), while $M_{\text{air}}=0.75 M_{\text{N}_2} + 0.23 M_{\text{O}_2} = 30 \text{ g/mol}$). The float pressure will be

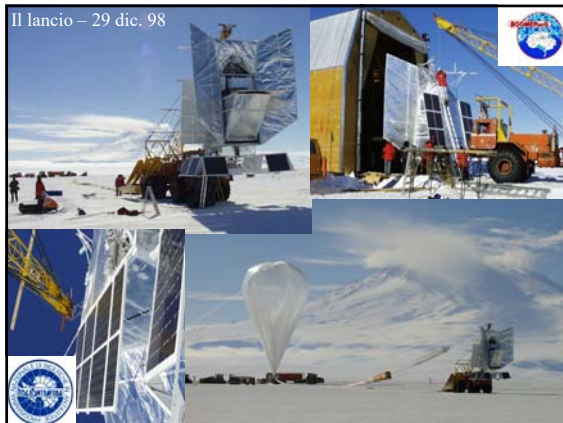
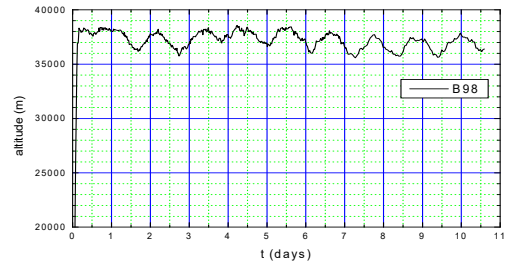
$$p_{\text{float}} = \frac{RT}{V} \frac{m_{\text{payl}} + m_{\text{ball}}}{M_{\text{air}} - M_{\text{gas}}} = \frac{0.0821 \times 230 (10^3 + 10^3) \times 10^3}{10^9 \quad 30 - 4} \text{ atm} = 1.5 \text{ mbar}$$

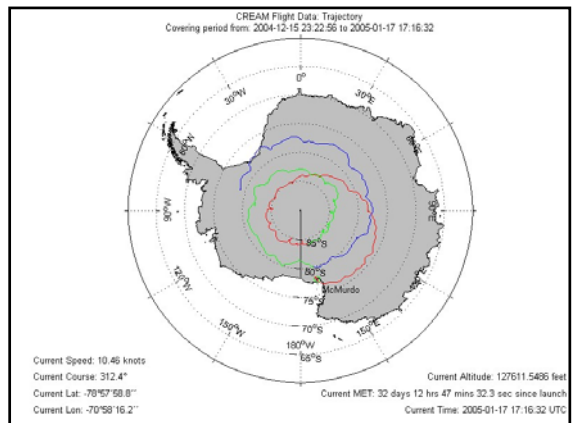
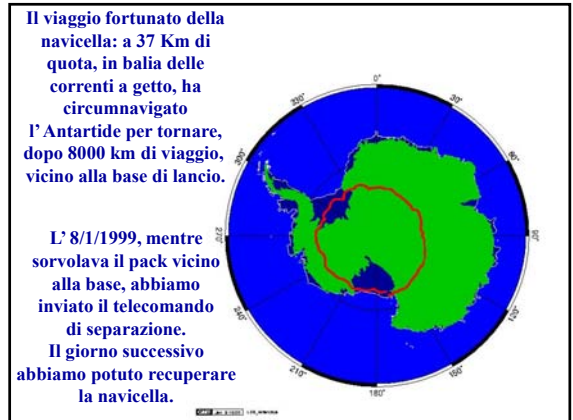
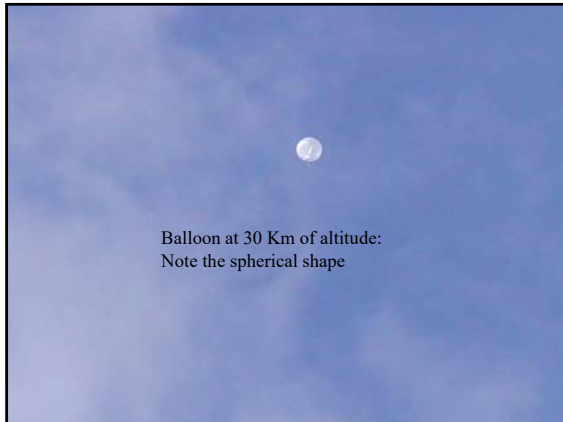
- Which corresponds to an altitude of about 44 Km.
- The cost of this balloon is around 150kEuro.





- The disadvantage of these balloons is that they can vent He when the temperature increases. So the volume decreases at each diurnal cycle. As a consequence, the float altitude decreases.
- Long Duration Balloon flights of a few weeks can be obtained in polar regions, where the diurnal illumination change is minimum. Example: The BOOMERanG flight in 1998, 10.6 days long:





Example 2

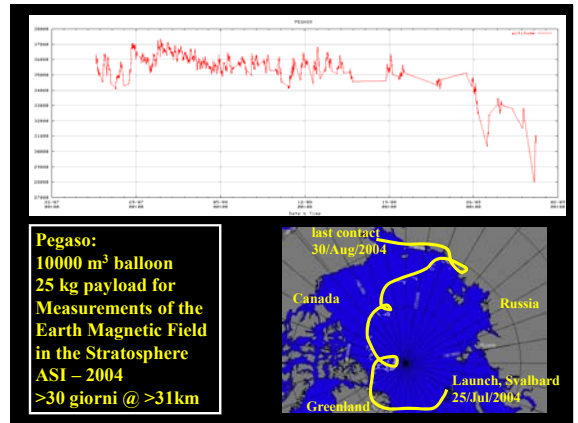
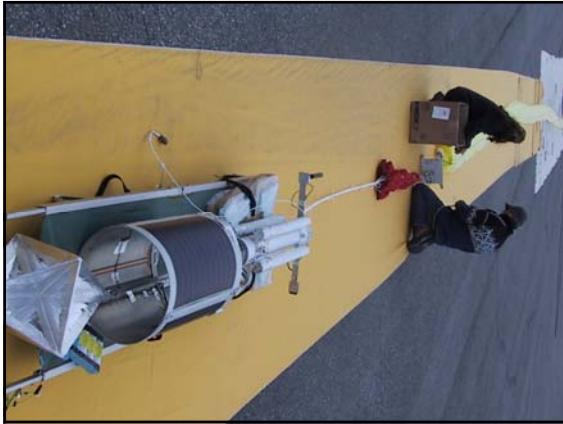
- Assume a 10^4 m^3 balloon (20 Kg mass) lifting a small 25 Kg payload. The balloon is inflated with He. The float pressure will be

$$P_{\text{float}} = \frac{RT}{V} \frac{m_{\text{payl}} + m_{\text{ball}}}{M_{\text{air}} - M_{\text{gas}}} =$$

$$= \frac{0.0821 \times 230 (25 + 20) \times 10^3}{10^7 \quad 30 - 4} \text{ atm} = 3 \text{ mbar}$$

- Which corresponds to an altitude of about 39 Km.
- The cost of this balloon is around 15kEuro.





Balloon Flights:

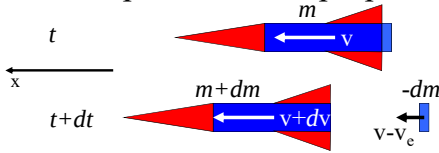
- ASI – Trapani – not anymore! Svalbard
- NSBF – Palestine Texas
- CNES – Aire Sur l' Adour
- China
- Russia
- Canada
- Brasil
- Japan

Principles of rocket propulsion

- The rocket works by expelling part of its mass at high velocity.
- By Newton's laws, the result is an increase of its speed.
- In this way the rocket enables a vehicle to be moved against the gravitational potential of the Earth, injected in orbit, or transferred from the gravitational field of one planet to that of another.



Principles of rocket propulsion



- Consider a rocket moving in vacuum in gravity-free space with speed v along the x axis.
- The rocket has mass m and ejects a mass dm in a time dt . So $dm < 0$. The speed of the ejected mass with respect to the rocket is v_e , in the negative x direction.
- Conservation of momentum for the system along x axis gives:

$$(m + dm)(v + dv) + (v - v_e)(-dm) = mv$$

$$m \cancel{v} + m dv + d\cancel{m}v + \cancel{m}dv - \cancel{v}dm + v_e dm = \cancel{m}v \Rightarrow m dv = -v_e dm$$

Principles of rocket propulsion

- Assuming a constant thrust (spinta) $T = v_e \frac{dm}{dt}$ i.e. a constant v_e , and constant mass flow rate, we get

$$dv = -v_e \frac{dm}{m} \rightarrow v - v_o = -v_e \int_{m_o}^m \frac{dm}{m} \rightarrow v - v_o = v_e \ln \frac{m_o}{m}$$

- Where m_o and v_o are initial mass and velocity of the rocket. When the rocket has exhausted all its fuel,

$$v_f - v_o = v_e \ln \frac{m_o}{m_f}$$

- The ratio m_o/m_f is called mass ratio. For a mass ratio larger than $e=2.718$, the final velocity of the rocket is larger than the ejection velocity. It must be maximized to have good performance.
- The fuel container / motor, and the payload need to be as light as possible with respect to the total fuel mass. The typical value of the mass ratio is around 5.

Principles of rocket propulsion

$$v_f - v_o = v_e \ln \frac{m_o}{m_f}$$

- The ejection speed v_e also has to be high.
- In chemical rockets the exhaust velocity depends on the heat energy liberated per unit mass, and on the molecular weight.
- To have a large ejection speed, the former should be as large as possible. The latter should be as small as possible.
- An important parameter in rocket design is the specific impulse I . This is defined as

$$I = \frac{v_e}{g} = \frac{T}{g \frac{dm}{dt}}$$

and has the dimension of time. For fluorine-hydrogen motors, this has a typical value of 300 s. The resulting v_e is around 2.5 km/s. So

$$v_f = v_o + v_e \ln \frac{m_o}{m_f} \cong (0 + 2.5 \ln 5) \text{ km/s} \cong 4.0 \text{ km/s}$$

- If we take into account the effect of gravity, v_f is even less.

Principles of rocket propulsion

- If the rocket is working in a gravitational field, the motion equation must be modified to account for gravity.
- Assume vertical motion. The change of momentum will be equal to the force times the time interval, i.e.

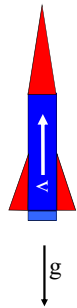
$$m dv + v_e dm = -mg dt \rightarrow dv = v_e \frac{dm}{m} - g dt$$

$$v - v_o = v_e \ln \frac{m_o}{m} - \int_0^t g(h) dt$$

- In the gravity field due to a planet of radius R_E

$$g(h) = g_E \left(\frac{R_E}{R_E + h} \right)^2$$

- Assuming a short time for ejection of all the fuel, so that the maximum altitude $h \ll R_E$, g can be considered constant.



Principles of rocket propulsion

- So we have

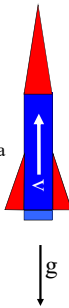
$$v_{fin} = v_e \ln \frac{m_o}{m_{fin}} - g t_{fin}$$

- The final velocity is higher if all the fuel is exhausted in a short time t_{fin} .
- For $t_{fin} = 100$ s, $v_{fin} = 3.0$ km/s.
- The altitude reached at t_{fin} can be computed assuming a constant fuel mass ejection rate:

$$\frac{dm}{dt} = -f \rightarrow m(t) = m_o - ft ; t_{fin} = \frac{m_o - m_{fin}}{f}$$

- during fuel ejection, the rocket reaches an altitude of

$$h(t) = \int_0^t \left(v_e \ln \left[\frac{m_o}{m(t)} \right] - g t \right) dt$$



Principles of rocket propulsion

$$\frac{dm}{dt} = -f \rightarrow m(t) = m_o - ft ; t_{fin} = \frac{m_o - m_{fin}}{f}$$

$$h(t) = \int_0^t \left(v_e \ln \left[\frac{m_o}{m(t)} \right] - g t \right) dt = v_e \int_0^t \ln \left[\frac{m_o}{m(t)} \right] dt - \frac{1}{2} g t^2 =$$

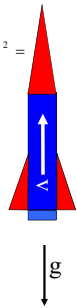
$$= \frac{m_o v_e}{f} \int_1^{m(t)/m_o} \ln \left[\frac{m}{m_o} \right] d \left[\frac{m}{m_o} \right] - \frac{1}{2} g t^2 =$$

$$= \frac{m_o v_e}{f} [x \ln x - x]_1^{m(t)/m_o} - \frac{1}{2} g t^2 =$$

$$= \frac{(m_o - m) v_e}{f} + \frac{m_o v_e}{f} \ln \frac{m}{m_o} - \frac{1}{2} g \left(\frac{m_o - m}{f} \right)^2$$

- For $t_{fin} = 100$ s, $v_{fin} = 3.0$ km/s, $h_{fin} = 137$ km
- After exhaustion of the fuel at t_{fin} , h_{fin} , the rocket will continue its flight in a ballistic, decelerated mode:

$$v_z = v_{fin} + a_z t = v_{fin} - g t ; t_{max} = \frac{v_{fin}}{g} ; h = h_{fin} + v_{fin} t - \frac{1}{2} g t^2$$



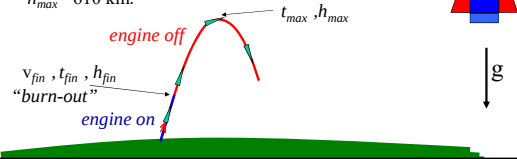
Principles of rocket propulsion

- For $t_{fin} = 100$ s, $v_{fin} = 3.0$ km/s, $h_{fin} = 137$ km
- After exhaustion of the fuel at t_{fin} , h_{fin} , the rocket will continue its flight in a ballistic mode:

$$t_{max} = \frac{v_{fin}}{g}; \quad h_{max} = h_{fin} + v_{fin} t_{max} - \frac{1}{2} g t_{max}^2$$

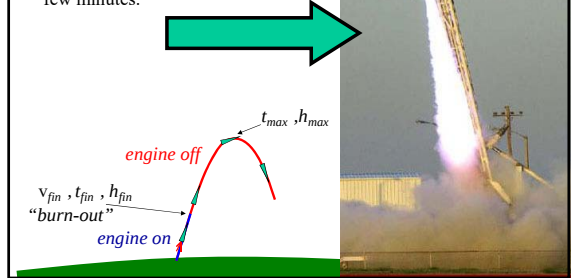
$$h_{max} = h_{fin} + \frac{1}{2} \frac{v_{fin}^2}{g}$$

- With the typical values above we get $t_{max} = 310$ s, and $h_{max} = 610$ km.



Sounding Rockets

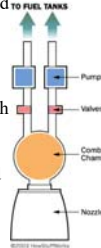
- As seen above, single stage sounding rockets can carry instrumentation at some hundred km of altitude for a few minutes.



Fuel needed

- In a chemical rocket motor, fuel and oxidizer are mixed in an exothermic reaction.
- The energy produced is converted (with some efficiency $\epsilon < 1$) into kinetic energy of the exhaust, which is ejected at high speed from a nozzle.
- For example, liquid H_2 can be used as fuel, and liquid O_2 as oxidizer, in the reaction

$$2H_2 + O_2 = 2H_2O + \Delta E$$
 where $\Delta E = 224800$ J/mol



- The total energy needed is $E = \int T dh = Th_{fin}$
- where the thrust is $T = v_e \frac{dm}{dt} = v_e \frac{m_o - m_{fin}}{t_{fin}}$

- so $E = Th_{fin} = v_e \frac{m_o - m_{fin}}{t_{fin}} h_{fin}$ *
- Here $m_o - m_{fin} = m_{H_2} + m_{O_2} = NW_{H_2} + \frac{N}{2} W_{O_2} = N \left(W_{H_2} + \frac{W_{O_2}}{2} \right)$ **

- The molecular weights W are 2 g/mol and 32 g/mol. The number of moles needed N will be simply $E/\epsilon \Delta E$, so combining with * and ** we estimate the needed efficiency of the engine:

$$\epsilon = \frac{h_{fin} v_e}{t_{fin} \Delta E} \left(W_{H_2} + \frac{W_{O_2}}{2} \right)$$

- With our typical numbers

$$\epsilon \approx \frac{137000 \times 2500}{228400} \left(0.002 + \frac{0.032}{2} \right) \approx 27\%$$

- i.e. the rocket engine has to convert about 30% of the chemical energy into kinetic energy of the ejecta.

Fuel needed

- The total mass available for fuel is

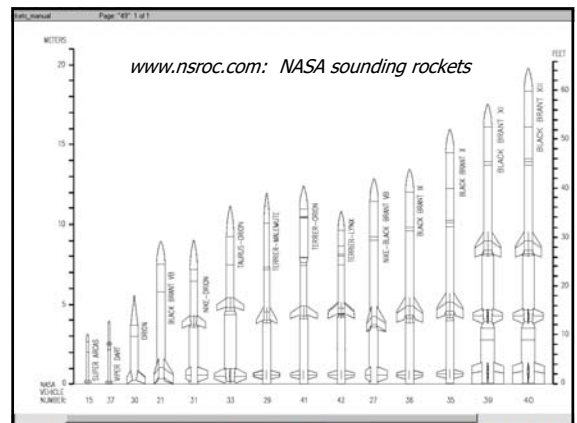
$$m_o - m_{fin} = (F - 1) m_{fin} = m_{H_2} + m_{O_2} \Rightarrow$$

$$(F - 1) m_{fin} = \frac{N}{2} W_{O_2} + N W_{H_2} \Rightarrow$$

$$N = \frac{(F - 1) m_{fin}}{W_{H_2} + W_{O_2} / 2} \approx \frac{4 \times 100}{0.002 + 0.032 / 2} \text{ mol} \approx 22200 \text{ mol}$$
- So we have

$$m_{O_2} = \frac{N}{2} W_{O_2} = \frac{22200}{2} \times 0.032 \text{ Kg} = 355 \text{ Kg}$$

$$m_{H_2} = N W_{H_2} = 22200 \times 0.002 \text{ Kg} = 44 \text{ Kg}$$
- These must be stored in liquid phase (cryogenics !), otherwise the volume of the containers would be too large. This means 628 liters of LH_2 (density 0.07 kg/l) and 309 liters of LO_2 (density 1.149 kg/l).
- The total chemical energy produced by this motor is $N \Delta E = 5 \times 10^9$ J and the thrust is $T = \epsilon N \Delta E / h_{fin} = 9800$ N.



Multi-stage rockets

$$M_o = (M_1 + m_1) + (M_2 + m_2)$$

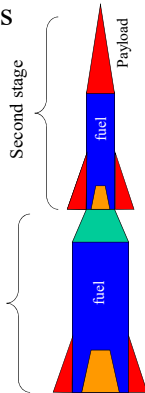
$$v_2 = v_{e1} \ln \frac{M_o}{M_o - m_1} + v_{e2} \ln \frac{M_o - (M_1 + m_1)}{M_o - (M_1 + m_1 + m_2)}$$

- Assume $R = \frac{M_1 + m_1}{M_1} = \frac{M_2 + m_2}{M_2}$

and define $x = \frac{M_2 + m_2}{M_1 + m_1}$

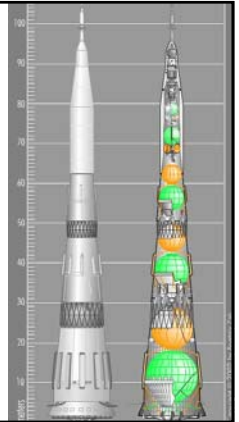
$$v_2 = v_{e1} \ln \frac{R(1+x)}{1+Rx} + v_{e2} \ln R$$

For a given R, this is maximum for $x \ll 1$, i.e. when the second stage is much lighter than the first one. For $x=0.1$ and for typical values of R and v_e , we get $v_2 = 13$ km/s, larger than the escape velocity from the Earth.



NL-L3

- Huge multi-stage rockets have been built for the most demanding space missions.
- The NL-L3 was a huge (100 m high) five stages russian rocket for moon-related missions ...



- And here is the russian fleet of rockets used the last 40 years, and being used today ...



NASA
Apollo 13
Saturn V

